Lidar Orbital Angular Momentum Sensor (LOAMS)
ACT-2013
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“The eye sees only what the mind is prepared to comprehend”
- Henri-Louis Bergson
Maxwell’s Equations – Basic EM Theory

• Maxwell’s equations describe the classical electromagnetic (EM) fields (vacuum)

\[ \nabla \cdot \mathbf{E} = 0 \]

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]

\[ \nabla \cdot \mathbf{B} = 0 \]

\[ \nabla \times \mathbf{B} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} \]

• These can be combined into wave equations that can be solved with different approximations e.g:
  — Different boundary conditions
  — Different coordinate system which implies different basis sets
  — Different approximations (e.g. plane wave)

• EM waves have the following properties
  — Transverse only – no longitudinal component
  — Wavelength (color), related to their frequency (vacuum) \( \lambda \nu = c \)
  — Polarization (axis for E field vector)
  — Carry Energy (Poynting Theorem)
  — Carry Linear Momentum (i.e. light exerts pressure)
  — Characterized by temporal and spatial coherence
  — Carry Angular Momentum
• Ideal solution (infinite extent)

\[ E(r,t) = (e^{\downarrow 1} E^{\downarrow 1} + e^{\downarrow 2} E^{\downarrow 2}) e^{\uparrow} - i(\omega t - kr + \phi) \]

Where \( \phi \) – constant, \( \omega = 2\pi v \), \( k = 2\pi/\lambda \)

• Similar result for B field

Linear Polarization

Circular Polarization
Different Solutions to EM Wave Equation - Gaussian

- Realistic solution for beams –
  - Lowest order solution to most laser cavities
  - Mode that occurs for NASA’s single frequency lasers (OAWL, HSRL, Twilite, DAWN, CATS .......)

\[
Gaussian(r, z,t) = E \cdot \frac{1}{w_{lo}} \exp\left(-\frac{r^2}{w_{lo}^2}\right) \cdot \exp(i[kz - \omega t + \varphi])
\]

Intensity Distribution

Cross-Section showing intensity versus radial position
Different Solutions to EM Wave Equation – Laguerre -Gauss

• Complete basis set for systems that have cylindrical symmetry
  — The lowest order mode (0,0) is just the Gaussian mode

\[
LG_{mp}(r,\varphi) = \frac{(2p!)}{\pi(|m|+p)!} \frac{1}{|m|!} \cdot \frac{1}{w_o} \cdot \left(\sqrt{2} \cdot \frac{r}{w_o}\right)^{|m|} \cdot L_p^{|m|} \left(2\frac{r^2}{w_o^2}\right) \cdot \exp\left(-\frac{r^2}{w_o^2}\right) \cdot \exp(i[m\varphi+kz-\omega t])
\]

Intensity distribution mode - (m,p)
Characteristics of the Laguerre-Gauss Solutions

- For \( m \neq 0 \), this represents an unexplored degree of freedom for the EM field.
- For \( m \neq 0 \), there is a null ("singularity") on axis – which gives the name "vortex beams" or "helical beams".
  - The vortex is a defining aspect.
  - The null (or singularity) is preserved as this type of beam propagates, as opposed to a beam where an obstruction blocks the center and the hole fills in over distance due to diffraction.
  - Mathematically the null occurs because the phase would have to take on all values at this point.
• $m$ (or sometimes $\ell$ or $q$ is used) is the “winding number” or the “topological charge” – it tells how many times the phase wraps around the axis as the wave propagates forward.

  Beam propagates forward in $z$ like a screw, with the phase now dependent on $z$, $t$: 
  \[
  \varphi(z,t) = \omega t - k z/m
  \]

$\varphi(z,t)$ is the phase of the wave at position $z$ and time $t$. The phase wraps around the axis $m$ times as the beam propagates forward in $z$. The pitch of the beam is given by $m/\lambda$, where $\lambda$ is the wavelength of the wave.
More Characteristics of Laguerre –Gauss Solutions

- Can have any polarization, but also carries a different form of angular momentum relative to its null axis – this is called Orbital Angular Momentum (OAM) to separate it from that carried by polarization
  - Instead of polarization it comes from the spatial phase structure
  - This affects interactions with matter because angular momentum must be conserved
  - Polarization is limited to +/- 1 unit of angular momentum, OAM is not limited
  - Demonstrated at the single photon level – fundamental (intrinsic) property of light
  - Demonstrated that OAM will exert a torque on particles (extrinsic property)
  - The angular momentum carried by polarization (SAM) and OAM are not always separable, but are in the paraxial approximation
Applications and Speculations

- Researchers are now modulating OAM values to transmit information
  - Free Space lasercom (impacted by turbulence)
  - Fiber based communication (requires special fiber)
  - Quantum entangled states for secure communication

- Optical tweezers
  - Now manipulating cells, small particles, trapped atoms by adding beams with OAM to optical microscopes – vortex beams apply torque causing particles to spin about beam axis

- Studies of Atmospheric Turbulence
- Transverse Doppler measurements (arising from the Poynting vector being skewed off-axis)
- Astronomy is using analogous techniques in “vortex coronagraphs”

Speculations (with many competing peer reviewed papers written on the subject):
- Light from natural sources will carry OAM which will enable new physical measurements
- The conservation of angular momentum will impact how beams carrying OAM interact with atoms, molecules and particles in non-intuitive ways
• Use the unique spatial coherency of the laser beam to create a new spatial filter that will
  — Allow us to relax stability requirements of traditional filter
  — Improve daytime filtering, reducing background noise
  — Reduce biases due to multiple scattering
  — Eliminate obscuration loss from telescope secondaries
  — (But what is the impact of speckle? Alignment requirements? Etc)

• Analyze the possibility that higher order L-G beams (including vector vortex beams) could have different interaction strengths with atmospheric constituents - i.e. modify Rayleigh, Mie, or extinction cross-sections
  — Look at this both experimentally and theoretically
1) Experimental Demonstrations of properties of beams and scattering properties
   — Led by Jeff Applegate
   — Start by building up the toolset to create and detect beams in the lab (SPP, SLMs, SH, mode sorters)
   — Study scattering properties of particles,
   — Study particle scattering in turbulent fields
   — Measure the ability to use spatial coherence to separate coherent and incoherent fields

2) Numerical simulation and modeling of optical system for OAM creation/detection
   — Led by Mike Lieber
   — Start by building a toolset of Matlab/Simulink models that match the lab instruments
   — Perform “numerical experiments” simulating the instrument light interaction
   — Completed models of vortex beams passing through turbulence
   — Detailed models of creation of vortex beams with spatial light modulators

3) Electromagnetic field modeling of OAM light interacting with aerosol particles
   — Led by Yong Hu and Wenbo Sun (NASA LaRC)
   — Requires re-assessment of assumptions in existing scattering models
   — Utilizes Finite Difference Time Domain electromagnetic field models
   — Completed Tet case of Mie scattering from single and pairs of particles
   — Combined OAM models with the idea of “photon sieves” flat diffractive optics showing that sieve will separate the coherent beam from the natural sunlight
1) Status of Lab Demonstrations
• Currently developing the experimental tools with guidance from 20+ years of research papers

• Started using Spiral Phase Plates to understand beam manipulation of L-G beams

![Beam images](m=0 Gaussian, SPP = +4)

![Beam images](m=+4 Gaussian, SPP = +4)

![Beam images](m=0 Gaussian, SPP = -4)

Beam into page Beam out of page
Early Results – Detection of OAM in beam using Shack Hartmann Interferometer

Poynting Vector Angle

\[ g = \frac{m}{kr} \]
2) Status of LOAMS Instrument modeling testbed
- Matlab/Simulink based, including Adaptive Optics Toolbox
- Leverages extensive history at Ball of Integrated modeling and optical wavefront analysis and control
- Baseline model now completed and is guiding next stages of lab development

For now is simply a reflector – need to replace with scatter model from Yong Hu

Ray trace input intensity, polarization, etc.

Wavefront errors, 6 DOF displacements

Refractive plate - need to add in phase

Add in spatial filter assembly
• Modeling Shack Hartmann measurement of OAM beam and systematic errors

Effect of choosing wrong (top) or right (bottom) centroid of vortex for analysis

Effect of having non-ideal transfer optics on the Shack Hartmann measurement (1 Wave of error on optics)
• Reconstructing the wavefront of the vortex beam measured on the Shack Hartmann using algorithms developed for adaptive optics for large telescopes

• Model the impact of refractive turbulence on a beam by using a phase plate, producing speckle

\[ \frac{D}{r_0} = 2 \quad \text{and} \quad \frac{D}{r_0} = 8 \]
3) Status of the Numerical Modeling of OAM beams and their scattering using Finite Difference Time Domain model
• Utilizing Finite Difference Time Domain modeling to study the evolution of EM fields
  • Grid-based differential numerical method for solving Maxwell’s equations for arbitrary boundaries and space
  • Uses a staggered grid, one for E and one for B, for each time step
  • Computationally intensive because of the grid size required (to match wavelength and scattering features – all at a distance), preliminary results are limited because of use of PCs – now moving to computer cluster at LaRC
Slice perpendicular to axis shows rotation with time.

Slice along axis shows:
- Free-space propagation
- Transmission through an $n=2$ window
Examples from Mie Theory – 532 nm and a 1 micron particle

http://omlc.org/calc/mie_calc.html
From Premoze Symposium on rendering 2004
Results of FDTD Model – Forward scattering is suppressed by when beam carries OAM which will change the multiple scattering

- Begun studying the scattering matrices for an idealized case of a single particle in a tightly focused beam
- Scattering at different angles as a function of particle size parameter $X$, and index $m$ and location, OAM mode order
Test Setup to validate results of scattering model

- OAM beam
- Lens assembly (f=50mm)
- Microspheres on rear surface of target window
- Unscattered beam
- Scatter
- Opaque mask to block unscattered beam
- Paper screen
- Camera
OAM sign comparison; $L=10$ vs. $L= -10$
(beam centered on peak intensity of ring)

- Changing the OAM content changes the tip of the wavefront (or skew angle) causing the scattering to shift more off axis
- Working to quantify this
We have a long way to go, but have made a good start

Thanks to ESTO for providing us this opportunity to work on this new challenge, which we hope will benefit future Earth Science Missions
Back-up
Example – How to use the OAM of the light to create a Spatial filter

Modeled after Swartzlander’s Coronagraph demonstration
First Year Highlights

- Theoretical analysis of Mie scattering for beams of arbitrary OAM
- Use a breadboard lab set-up to generate and detect lasers with OAM and their scattering properties for controlled particle sizes and concentrations
- Demonstrate background rejection using heliostat – what is OAM content of natural scenes?
• Build a Brassboard mode sorter based on results from first year
• Test Mie scattering in a turbulence generator tank – OAM mode coupling
Peering into darkness with a vortex spatial filter

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PHYSICAL REVIEW A 86, 053830 (2012)

Mie scattering of purely azimuthal Laguerre-Gauss beams:
Angular-momentum-induced transparency

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(Received 12 July 2012; published 27 November 2012)

Orbital Angular Momentum Exchange in the Interaction of Twisted Light with Molecules


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3School of Chemistry, University of East Anglia, Norwich, NR4 7TJ, United Kingdom
(Received 29 April 2002; published 11 September 2002)
Examples of some research in the field

Backscattered polarization patterns, optical vortices, and the angular momentum of light

Chaim Schwartz and Aristide Dogariu
College of Optics, University of Central Florida, Orlando, Florida 32816

Enhanced backscattering of vortex waves from volume scattering media

Chaim Schwartz, Aristide Dogariu
College of Optics and Photonics, University of Central Florida, 4000 Central Florida Blvd., Orlando, FL 32816, United States

Taking the vector vortex coronagraph to the next level for ground- and space-based exoplanet imaging instruments:
review of technology developments
in the USA, Japan, and Europe

Dimitri Mawet, a,b, Naoshi Murakami, c, Christian Delacroix, d, Eugene Serabyn, b, Olivier Absil, d,
Naoshi Baba, c, Jacques Baudrand, c, Anthony Boccaletti, c, Rick Burruss, b, Russell Chipman, b,
Pontus Forsberg, c, Serge Habraken, d, Shoki Hamaguchi, c, Charles Hanot, d, Akiyoshi Ito, c, Mikael Karlsson, c, Brian Kern, a, John Krist, b, Andreas Kuhnert, b, Marie Levine, b, Kurt Lieber, b, Stephen Mcclain, f, Scott Mceldowney, b, Bertrand Mennesson, b, Dwight Moody, b, Hiroshi Murakami, c,
Albert Niessner, b, Jun Nishikawa, a, Nada O' brien, k, Kazuhiko Oka, c, Peggy Pack, b, Pierre Piron, d,
Laurent Pueyo, f, Pierre Riaud, f, Moritsugu Sakamoto, c, Motokide Tamura, a, John Trauger, b,
David Shemo, b, Jean Surdej, d, Nelson Tabiryan, m, Wesley Traub, b, James Wallace, b, Kaito Yokoi, n
Examples of some research in the field

Detection of a Spinning Object Using Light's Orbital Angular Momentum
Martin P. J. Lavery et al.
Science 341, 537 (2013);
DOI: 10.1126/science.1239936

Propagation of vector vortex beams through a turbulent atmosphere
Wen Cheng, Joseph W. Haus and Qiwen Zhan*
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Terabit-Scale Orbital Angular Momentum Mode Division Multiplexing in Fibers
Nenad Bozinovic et al.
Science 340, 1545 (2013);
DOI: 10.1126/science.1237861
Appendix #2
Historical Aspect
The Wave Motion of a Revolving Shaft, and a Suggestion as to the Angular Momentum in a Beam of Circularly Polarised Light

J. H. Poynting

First demonstration that EM field carries Angular Momentum - Via Polarization (SAM)

Mechanical Detection and Measurement of the Angular Momentum of Light

Richard A. Beth,* Worcester Polytechnic Institute, Worcester, Mass. and Palmer Physical Laboratory, Princeton University
(Received May 8, 1936)

Polarized beam of light passing through a stack of waveplates suspended by thread in a vacuum was found to exert a torque that rotated the waveplate stack, as proposed by Poynting.
• In conclusion, it has been demonstrated that absorptive particles trapped in the dark central minimum of a doughnut laser beam are set into rotation. The rotational motion of the particles is caused by the transfer of angular momentum carried by the photons. Since the laser beam is linearly polarized, this must originate in the "orbital angular momentum associated with the helical wave-front structure and central phase singularity. We have shown that the direction of the rotational motion is determined by the chirality of the helical wave front.