Low Energy Motions in the Earth-Moon System, Chaos, and Weak Capture

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Abstract - A main result that has been recently obtained is a new and deeper understanding of a region about the Moon where weak capture occurs, with significant applications. This region has been known to exist since 1986 by this researcher, and has important applications for its use in obtaining a new type of transfer to the Moon, using ballistic lunar capture(i.e. no Delta-V). One of these transfers was demonstrated by this researcher in 1991 with the rescue of the Japanese lunar spacecraft *Hiten*. Another was also used in 2004 for ESA's *SMART-1* spacecraft. However, a deep understanding of this region has been elusive. Now, through this research effort presented in this paper, the nature of the weak capture region, called the weak stability boundary, has been found. It is shown to support only resonance motions about the Earth, in resonance with the Moon, which chaotically transition from one type to another. This is a surprising observation and significant. It has been uncovered through the use of special visualizations on so called Poincare surfaces of section which have an exquisite geometric structure. The weak stability boundary itself has been shown to be an interesting dynamic region existing within a chaotic sea about the Moon in the phase space. These results have already led to an understanding of a transfer type: ballistic escape from the Earth-Moon system. First observed in 1990, they were not understood. Now, much more is known about their dynamics. These transfers may have potentially important applications, especially for future Mars missions. New types of low energy orbits are also described about the Moon which enable inclination changes by a factor of 12 less Delta-V than traditional orbits, enabling new lunar architectures in support of a lunar base.

I. INTRODUCTION

A new type of transfer from the Earth to the Moon was discovered by Belbruno and used in 1991 to enable a Japanese spacecraft, *Hiten*, to successfully reach the Moon. This transfer has the distinguishing property that it uses ballistic capture near the Moon., sometimes referred to as a ballistic capture transfer [1,2]. This means that the spacecraft goes into orbit about the Moon automatically, without the use of rockets to slow down. This is unlike the more traditional Hohmann transfer that needs to use a substantial amount of fuel from it's rocket engines to achieve lunar orbit. For this reason, the ballistic capture transfers are called low energy. The *SMART-1* spacecraft of ESA used another type of ballistic capture transfer in 2004 [2]. The transfer used by Hiten is particularly important since it's time of flight is only about 3 months, and it can deliver twice as much payload into lunar orbit as compared to the Hohmann transfer, for the same cost. For the US return to the Moon to construct a lunar outpost, such a transfer would be important for cost reductions for massive payloads not requiring human passengers.

It turns out that ballistic capture is equivalent to something called 'weak capture', which is a form of capture at the Moon, or any other secondary body orbiting a larger one (e.g. Europa orbiting Jupiter), where the two-body energy, E, of a spacecraft(SC), w.r.t. the Moon is negative(or zero) and where the capture is temporary. This means that the motion about the Moon is unstable. The spacecraft will be captured about the Moon for a short amount of time, then it will escape the Moon. In the weak capture state, the spacecraft is in the transitional boundary between capture and escape. The energy E is less than zero for a finite period of time. One may think that this form of capture is not good for applications since it is unstable – but it turns out to be ideal, since only a tiny amount of stabilization Delta-V is needed to achieve a relatively stable orbit [1,2]. This is carefully studied in [4]. Thus, understanding where weak capture can occur about the Moon is very important. This region was mapped out by Belbruno in 1986 [1,2]. It is called a weak stability boundary, and is a region in position-velocity space. It can be thought of as a generalization of the Lagrange points. These are five locations about the Moon where a spacecraft, that is fixed w.r.t. the Earth and Moon, will *remain* fixed there. In particular, the forces of gravity on the spacecraft will be balanced together with the outward centrifugal force. However, in the weak stability boundary, these forces will be balanced for a *moving* spacecraft. So, instead of just getting five points, one gets a complicated region. A spacecraft in this region is weakly captured by the Moon. Ballistic capture transfers from the Earth to the Moon are obtained by finding a transfer that goes to the weak stability boundary near the Moon. To be in this weak capture region at a particular location near the Moon, the spacecraft has to be moving with a critical velocity.

The WSB is very good for applications, but understanding its dynamic mathematical nature has been very difficult since this region gives rise to chaotic motions where the trajectory is very sensitive and unstable. Although one can locate the approximate location of this region with the computer, getting a deeper understanding of it is much more challenging. One fact was discovered in 1990 by Belbruno by numerical simulations: When a trajectory is in weak capture near the Moon, then in forward time and backward time, the spacecraft transitions onto an elliptic type orbit about the Earth that was in resonance with the Moon. That is, if the period of motion of the SC about the Earth is TS and the period of the Moon is TM, then mTS = nTM, where m,n are positive integers(1,2,3….). We call these 'm:n orbits' – which means that when SC goes around the Earth m times, the Moon goes around the Earth n times. Thus, the spacecraft would do a 'hop', or resonance transition, from one resonance orbit to another via weak capture.

This motion was found to be analogous to that of comets which are m:n orbits about the Sun, in resonance with Jupiter, and this led to a paper by Belbruno and B. Marsden in 1997 to study the WSB of Jupiter for such comets [2]. In all cases, it was found that such resonance hopping comets passed through weak capture near Jupiter in it's WSB. (Examples of such comets are Gehrels 3 or Oterma which hop between 2:3 and 3: 2 orbits.) Work by Koon et. al. in 2000 found numerically that a complicated network of dynamic channels (shaped like tubes) played a role in the hop process at Jupiter [8]. Such a network of tubes was conjectured by Belbruno in 1994 to be associated with the WSB at the Moon, under restrictive conditions, and associated with the *Hiten* transfer [1]. Beyond this work, little was known of the WSB and in particular, its connection with resonance motions.

In Section II, we describe a way to visualize the WSB about the Moon, under general conditions, and see that it is intimately associated with hop motions from one resonance to another by visualizing special slices of the position-velocity space, called Poincare sections. It is generally found that the WSB exists within a chaotic sea of points where there exists islands of invariant curves, of invariant tori, associated to resonance motions. The correlation dimension is also mentioned. Applications are discussed in Section III to Mars missions using low energy escape transfers and for special low fuel orbital motion about the Moon. This work is supported by the NASA AISR program.

II. VISUALIZATION OF THE WEAK CAPTURE REGION

We very briefly describe the visualization of the Poincare sections, to just give an idea of how it is done, and show some main results. The details are contained in [5].

The model we use to numerically simulate the motion of the spacecraft is given by the planar circular restricted three-body problem. This defines the motion of SC, of zero mass, on the plane of motion of the Earth and Moon, which are assumed to move in purely mutual circular orbits about their common center of mass, approximately at the Earth itself. We use a rotating coordinate system, (x,y), which rotates uniformly with the Earth and Moon, where the x-axis is long the Earth-Moon line. The coordinates are scaled so that the Earth is at the location (-m, 0), and the Moon is at (1-m, 0), where m=.0123 is the approximate mass ratio of the Moon to the Earth. In this system the Earth and Moon are fixed. The differential equations for the motion of SC are well known in the literature(See [1]). They can be written as a second order system (i.e. second order derivatives w.r.t. time, t,) of two differential equations for x, y:

DDx = F(dx/dt,dy/dt,x,y),
DDy = G(dx/dt,dy/dt,x,y),

where D=d/dt, and F,G are functions of dx/dt,dy/dt,x,y,. We let Z = (x,y,dx/dt,dy/dt), and Z(t) be a general solution of the differential equations, where t is defined over the interval [t1,t2], t1 < t2. Associated to this system is the general energy of SC, called the Jacobi energy function, J(Z), which is a constant along solutions, J(Z(t)) = C. C is called the Jacobi constant. The motion of SC can be restricted to the surfaces Q = {J =C}. which are three-dimensional in the four-dimensional phase space Z, where Q = Q(C).

To view the dynamics of SC to get a global understanding of it's motion, we make special two-dimensional slices through Q, for different values of C. We choose C < C1, where C1 = 3.184163, the value that J has at the exterior collinear Lagrange point w.r.t. the Moon, L1. For C < C1, the Hill's region of allowable motion of SC implies that SC is minimally energetic enough to escape the Earth-Moon system. As C decreases from C1, SC becomes increasingly energetic. We are interested in decreasing C and seeing what happens to the global dynamics of SC, on the special two-dimensional slices through Q. These slices are defined by normalizing them to the 2:1 resonance orbit initial conditions Z(0), which turn out to be a function only of the osculating eccentricity e of SC. The two-dimensional section is labeled S and defined where S = {y = 0, Dy > 0}. These all called Poincare sections, and the flow of the differential equations is transversal to S = S(C).

We now look at some pictures of S(C), in suitable coordinates, and see what the dynamics is for SC. To make them, hundreds of thousands of intersections are made with the numerically integrated trajectory of SC on S, and the resulting points of intersection are recorded. After this is done, complex patterns emerge showing the dynamics of SC. The first one is for C = 3.181768 shown in Figure 1. This corresponds to e = 0. One sees resonance islands for different resonances: 4:1, 5:2, 7:3, 2:1. The islands consist of invariant curves formed from invariant tori. The many points seen on the section without a clear pattern, define chaotic motion for SC, and collectively are called a chaotic sea. A blowup of part of this section is seen in Figure 2, showing the 2:1 resonance island structure.

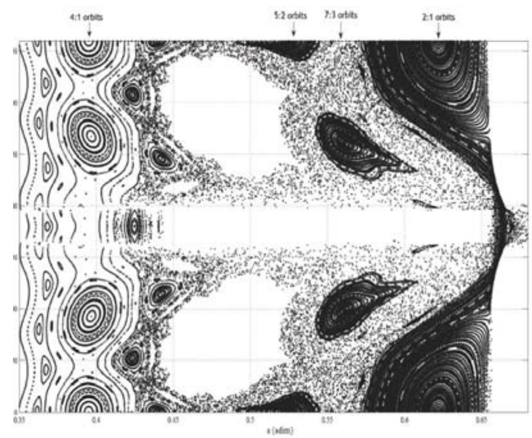

Figure 1 Section S for C = .1811768. Resonance islands are shown for 2:1, 4:1, 5:2, 7:3 orbits.

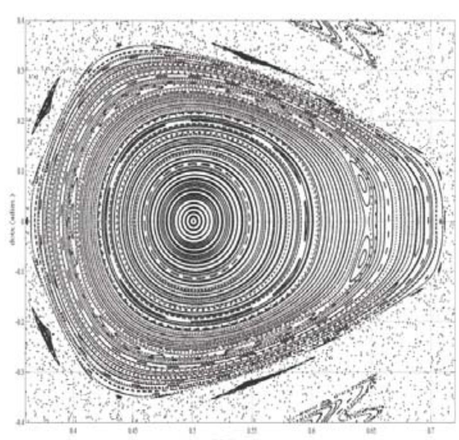

Figure2 Magnification of 2:1 resonance island.

The chaotic sea proves that SC can move between resonance islands freely, and perform resonance hops. This is an interesting result. The invariant curves form an extremely complicated invariant manifold network in the full phase space Z of infinitely many interwoven tubes.

As C is further decreased the section changes considerably. In Figure 3, C = 2.734978, corresponding to e=.7. In this figure, the chaotic sea has increased substantially, and the range of the hop motion is much larger, with fewer resonance types. It is interesting to note that only resonance hops seem to occur on S for a range of C [5].

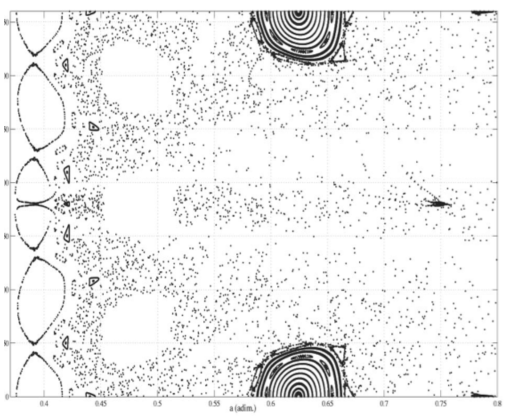

Figure 3 Section for C = 2.734978

Many more sections are described in [5], and collectively the dynamics of SC is understood w.r.t its resonance motion.

We conclude this section with a visualization of the WSB. Using the definition of the WSB from [1], which, on energy surfaces Q, we satisfy H < 0 (or =0), and d(rm)/dt = 0, where rm is the distance from SC to the Moon , we find its extension with d(rm)/dt > 0, shown in Figure 4. The WSB itself lies on the horizontal x-axis through the middle of the plot that the extended WSB projects onto.

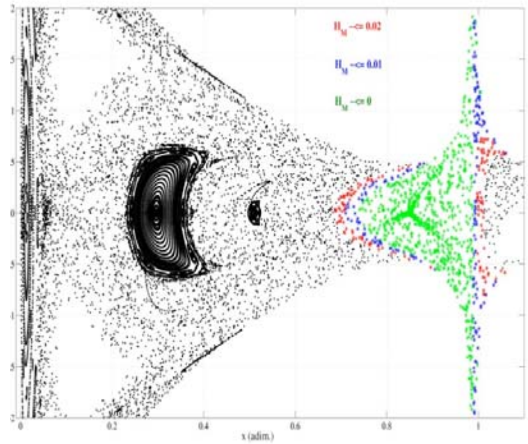

Figure 4 The extended WSB for C = 2.973425. The green points with HM = E < 0 represent the extended WSB. It exists within a chaotic sea near the Moon. This sea is allows SC to hop between different resonances and near the WSB.

This is the first time this region has been visualized, and its complexity shows how closely resonance motion is associated to ballistic capture and the lunar transfer used by *Hiten* and *SMART-1*, as well as comets discussed earlier. It gives insights into intriguing motions for important mission applications.

In [5], the correlation dimension is defined and used to estimate the fractal nature of the WSB, which gives further insights.

III. APPLICATIONS

The ballistic capture transfer to the Moon, has proven itself to be a revolutionary approach to travel to the Moon, already having been demonstrated by two spacecraft, *Hiten*, and *SMART-1*. It's ability to transport more mass to the Moon for about half the usual cost promises to be an important consideration for future missions to the Moon, especially for the construction of a lunar outpost. The WSB enables these transfers to exist, and the results described in this paper, and more generally in [5], show that the WSB is closely tied together with resonance motions about the Earth in resonance with the Moon. Associated to these resonances is a complicated network of dynamic channels which weave throughout the phase space between the Earth and Moon. Although it has been known that dynamic channels exist for low energy transfers, their close connection to resonance behavior is new, as presented here. This gives an alternate perspective on such motion, and offers a promising new direction of study to pursue.

The association of the WSB to resonance motions and the hop, in particular, between different resonance orbits about the Earth can be used not just for the purpose of weak ballistic capture at the Moon, but for ballistic escape from the Earth-Moon system, where no Delta-V is needed. This is discussed briefly in [2,5], and is currently being researched. However, the idea can be presented at this point. As we have discussed, if the gravity of the Sun is not modeled, then when a spacecraft is going around the Earth in an m:n orbit and encounters the WSB at the Moon for temporary weak capture, it generally exits onto another resonance orbit, say a i:j orbit, staying relatively close to the Earth-Moon system. However, when the Sun's gravity is modeled, it has the effect of causing the m:n orbit not to transition onto a bounded i:j orbit, but rather an unbounded trajectory which escapes the Earth-Moon system, after about 100 days. That is, the Sun's gravity pulls the i:j orbit apart into an unbounded trajectory which, after escaping the Earth-Moon system, goes into an Earth-like orbit about the Sun. One of these ballistic escape trajectories is seen in Figure 5. The spacecraft starts out in a 2:1 orbit, encounters the Moon's WSB, then ballistically escapes the Earth-Moon system.

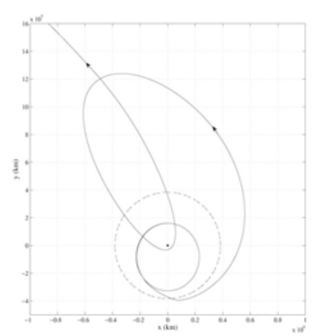

Figure 5 A ballistic escape trajectory going from a 2:1 to an escape from the Earth-Moon system with no Delta-V.

The very interesting observation is that when SC goes into an Earth-like orbit about the Sun, it turns out to also be a resonance orbit, but now in resonance with the Earth's motion about the Sun. These ballistic escape transfers from the Earth, save about 1 km/s in Delta-V, which can be used to send payloads to Mars, after an additional Earth flyby, for much less propellant. Such payloads could be used as supply depots for missions on the way to Mars, or just to deliver payloads to Mars itself. These ballistic escape transfers were first noticed by Belbruno in 1990 [2], but their connection to resonance motions is now uncovered, which should open up their utility for applications.

It is finally mentioned that the existence of very low energy orbits about the Moon was described in [4]. The results were obtained originally from an AISR project in 2006. These orbits can orbit the Moon for extended periods of time and change their inclination for a factor 12 less Delta-V than before, thus being potentially important for lunar mission architectures to deliver payloads into lunar orbit and onto the surface of the Moon at a substantially lower cost. They can also be used to increase the time in orbit of a lunar satellite for less fuel. Such orbits have a high eccentricity and large apoapsis w.r.t the Moon, of about 40,000 km. They can stay in lunar orbit for long periods of time in order to also serve as an ideal communication system with the Earth and all points on the lunar surface. This offers a viable and less expensive alternative to using halo orbits about the collinear lunar L1, L2 points for a communication system, see [4].

ACKNOWLEDGEMENT

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